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CRITERIA FOR THE DESIGN OF PRESSURE TRANSDUCER ADAPTER SYSTEMS

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NOMENCLATURE

a = Radius of tube 2 [in]
c = Speed of sound [in/sec)
f = Frequency (Hz)
k = ω/c = Wave number [rad/in]
L_1 = Length of tube 1 [in]
L_2 = Length of tube 2 [in]
P_1 = Pressure in tube 1 [lb/in^2]
P_2 = Pressure in tube 2 [lb/in^2]
P_f = Pressure at transducer face [lb/in^2]
P_m = Pressure at measurement location [lb/in^2]
Q_1 = Volume velocity in tube 1 [in^3/sec]
Q_2 = Volume velocity in tube 2 [in^3/sec]
R = V_1/V_2 = Volume ratio
S_1 = Area of tube 1 [in^2]
S_2 = Area of tube 2 [in^2]
t = Time [sec]
V_1 = S_1L_1 = Volume of tube 1 [in^3]
V_2 = S_2L_2 = Volume of tube 2 [in^3]
x_1 = Coordinate of tube 1 [in]
x_2 = Coordinate of tube 2 [in]
ω = $2\pi f$ = Frequency [rad/sec]
ρ = Density of gas [lbm/in^3]

INTRODUCTION

The importance of transducer frequency response in measuring dynamic pressures is well recognized and is reflected in the catalogues of transducer manufacturers. The value of the highest frequency until which the transducer will transmit information without either magnification or attenuation of more than a nominal amount is communicated to the customer. This frequency is usually a certain fraction of the first natural frequency of the measuring device. It is also generally recognized that a further reduction in frequency response takes place if the transducer face is not flush mounted at the measurement location but set back a certain distance, connected to the measurement location by a small volume, orifice or narrow channel, or a combination of these. In many application cases, with modern transducers like the piezoelectric type with high cut-off frequencies of 100 kHz or more, the mounting scheme is actually the limiting factor of frequency response. It is, therefore, of importance to compute the frequency response of such mounts with a certain degree of accuracy. In doing this, over-simplifications should be avoided. For example, simple Helmholtz resonator theory is generally not satisfactory for many practical adapter systems with a long connecting tube or passageway. Also, if the adapter connecting tube is modeled as a simple organ pipe, errors will result due to the neglection of the small volume at the transducer location. Thus, a theory which is valid for both of the above cases would eliminate the possibility of over-simplification.

For compressor applications, geometrical considerations sometimes necessitate the use of adapter systems. The measurement of compressor cylinder pressure, for example, generally does not allow the transducer diaphragm to be flush mounted with the cylinder wall. In this case an adapter system of small overall volume may be required to avoid a significant change in the compressor clearance volume. Transducer adapter systems may also be required for the measurement of pressure in the suction and discharge plenums. However, for this application, frequency components associated with standing waves in the plenums may excite a resonance condition in the adapter. Since these standing wave frequencies may be very high, it is difficult to design an adapter which will not be affected by one of the numerous frequency components present in the plenums. Thus, if at all possible, adapter systems should not be employed for pressure measurements in plenums or other fixed geometry environments.

In the following, simplified frequency equations are derived from a general wave equation model and limitations of these equations are pointed out. A practical case is investigated numerically to point out errors that may typically occur by a too simple minded approach to the mounting problem. Experimental data using this adapter system is then presented to illustrate adapter resonances which may occur in compressor plenum pressure measurements.

It is believed that the presented exposition is not available in the literature in such a collected form, even while some of the derived equations are available, (Reference 2-4). It is also believed that the presented approach of deriving all equations of interest from a basic wave equation model has certain special merits as far as the basic understanding of Helmholtz resonator type dynamics is concerned.

THE BASIC SYSTEM AND ITS WAVE EQUATION SOLUTION

The basic transducer mount or adapter considered here consists of two concentric circular tubes connecting the measurement location (P_m) to the transducer face (P_f) as shown in Figure 1. It is assumed that the volume associated with the pressure to be measured is large compared to the total volume of the transducer adapter. This is true for any well designed measuring scheme. Thus, only frequency response criteria need be considered. It is also assumed that the transducer is of the piezoelectric type which typically has a frequency response much higher than that of a typical adapter. Thus, attention has to be focused on the adapter system since its frequency response will control the frequency response of the entire measuring set-up. For types of transducers that have a frequency response of the order of magnitude of the response of the adapter, transducer and adapter have to be investigated as a joined system.

To analyze the adapter system, plane wave acoustic theory is utilized. From a steady state solution to the wave equation (Reference 1), the following equations are obtained for pressures P_1, P_2, and volume velocities Q_1, Q_2.

$$P_1(x_1,t) = i\omega\rho c(A_1 e^{-ikx_1} + B_1 e^{ikx_1})e^{i\omega t} \quad (1)$$

$$P_2(x_1,t) = i\omega\rho c(A_2 e^{-ikx_2} + B_2 e^{ikx_2})e^{i\omega t} \quad (2)$$

$$Q_1(x_1,t) = i\omega S_1(A_1 e^{-ikx_1} - B_1 e^{ikx_1})e^{i\omega t} \quad (3)$$

$$Q_2(x_2,t) = i\omega S_2(A_2 e^{-ikx_2} - B_2 e^{ikx_2})e^{i\omega t} \quad (4)$$

The following four boundary conditions apply:

$$Q_1(0,t) = 0$$
$$Q_1(L_1,t) = Q_2(0,t)$$
$$P_1(L_1,t) = P_2(0,t)$$
$$P_2(L_2,t) = 0$$

Using Equations 1-4 to satisfy the above boundary conditions yields four linear homogeneous algebraic equations in terms of A_1, B_1, A_2, B_2.

$$\begin{bmatrix} e^{-ikL_1} & e^{ikL_1} & -1 & -1 \\ S_1 e^{-ikL_1} & -S_1 e^{ikL_1} & -S_2 & S_2 \\ 0 & 0 & e^{-ikL_2} & e^{ikL_2} \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \end{bmatrix} = 0$$

For the above system of equations to have a nontrivial solution, the determinant of the matrix of coefficients must be zero. Upon satisfying this criteria, the following frequency equation results.

$$\tan kL_1 \tan kL_2 = S_2/S_1 \quad (5)$$

The lowest frequency given by this equation represents the first resonance of the adapter system and should be well above the highest frequency of interest in the measured pressure signal.

THE HELMHOLTZ RESONATOR

Equation 5 is valid for all types of passage configurations which may be represented by two concentric tubes. For certain tube combinations it can be simplified to yield the classical Helmholtz resonator equation which is traditionally derived from simplified physical considerations. To do this, the tube dimensions must be assumed such that $kL_1 << 1$ and $kL_2 << 1$. Then,

$$\tan kL_1 \approx kL_1 \quad \text{and} \quad \tan kL_2 \approx kL_2$$

Equation 5 now becomes, after some manipulation,

$$f = \frac{c}{2\pi L_2} 1/\sqrt{R} \quad (6)$$

$$\text{where } R = \frac{V_1}{V_2} = \frac{S_1 L_1}{S_2 L_2}$$

Equation 6 is an approximate equation and is only valid for large values of the volume ratio, R. For example, Equation 6 will be greater than 5% in error (relative to Equation 5) for $R < 3$. Since most well designed adapter systems have a volume ratio of 1 or less, the Helmholtz equation should be avoided for this application.

COMMON ADAPTER CONFIGURATIONS

For many common adapter systems, $L_1 << L_2$, and only the assumption, $kL_1 << 1$, may be made for the general system defined by Equation 5. For this case, Equation 5 becomes:

$$(kL_2)\tan(kL_2) = \frac{1}{R} \tag{7}$$

This equation is valid for all values of R covered by the Helmholtz equation as well as for $R \to 0$. Note that at $R = 0$, Equation 7 may be expressed in the form:

$$\cos(kL_2) = 0 \tag{8}$$

From this equation the so-called organ pipe frequencies are obtained.

$$f = \frac{(2n+1)}{4}\ \frac{c}{L_2} \quad (n = 0,1,2,\ldots) \tag{9}$$

For adapter systems, Equation 9 can only be exact when $V_1 = 0$, and therefore, should not be employed except for very small values of R. For example, with $R > 0.1$, an error greater than 10% (relative to Equations 5 or 7) will result if Equation 9 is employed.

Equation 7 is recommended for the design of pressure transducer adapter systems. However, in its present form, the desired adapter resonant frequency may only be obtained by trial and error or iterative procedures. Fortunately, approximate equations are possible which allow an explicit calculation of the fundamental adapter frequency. If appropriate trigometric series approximations are made to Equation 7, the following approximating equation may be obtained for the first natural frequency of the adapter system. (See also Reference 2.)

$$f = \frac{c}{2\pi L_2}\ 1/\sqrt{R+0.5} \tag{10}$$

With this equation, errors as great as 10% are still possible for small values of R. To improve this result further, the constant, 0.5, was adjusted until a maximum error of 1.6% (relative to Equation 7) was obtained over the volume ratio range, $0 \leq R \leq 100$. The result is given below.

$$f = \frac{c}{2\pi L_2}\ 1/\sqrt{R+.3905} \tag{11}$$

From the above equation it is apparent that a reduction in either R or L_2 will serve to raise the first natural frequency of the adapter system.

In applying Equation 11 as well as other equations considered herein, a correction to the length, L_2, should be considered. Due to gas inertia at the connecting tube exit, $X_2 = L_2$, the length, L_2, should be increased by the factor $\pi a/4$, (Reference 4). In doing this, it should be remembered that R is also a function of L_2. For adapter systems the above correction is often insignificant since it is not uncommon for L_2 to be much greater than a.

ADAPTER DAMPING

If a resonant frequency of the adapter system is excited, gas pressure oscillations will occur which may introduce serious errors in the desired measured pressure. To minimize these errors, the adapter design should incorporate adequate damping. Two such methods for accomplishing this are suggested by Nagao and Ikegami (Reference 3): 1) an extreme narrowing of the adapter connecting tube and, 2) the installation of a small orifice at the connecting tube end, $X_2 = L_2$. With the first method, the volume ratio, R, must be held constant to maintain a constant value for the undamped natural frequency of the absorber system. Significant reductions in adapter frequency response will, however, still occur due to the increased damping in the system. With the second method adequate damping is obtained without a significant loss in frequency response. Nagao and Ikegami give an equation for calculting the optimum orifice to tube area ratio and agreement with experiment is shown.

A TYPICAL APPLICATION EXAMPLE

To demonstrate the varied results which may be obtained when different assumptions are employed in estimating resonant frequencies for transducer adapters or mounts, frequency calculations were made for a commercially available adapter used with piezoelectric transducters. This adapter was then employed in the measurement of a compressor discharge plenum pressure and pressure oscillations corresponding to adapter resonances were found to exist. The adapter considered had the following dimensions:

$S_1 = 0.0380\ in^2$
$L_1 = 0.0130$ in
$S_2 = 0.001272\ in^2$
$L_2 = 0.540$ in
$\pi a/4 = 0.016$ in
$R = 0.728$

The medium was Freon 12 with a speed of sound of $c = 5700$ in/sec.

Approaching the problem superficially, one may have been tempted to use the organ pipe solution, Equation 9, since the ratio L_2/L_1 was more than 40. However, the influence of the small volume above the transducer head is considerable, as the results given below reveal.

Equation	Frequency (Hz)
5	1582, 5740
7	1582, 5740
11	1557
6	1941
9	2563

Several frequencies are given for Equations 5 and 7 to illustrate the fact that many resonances are possible even though only the first natural frequency is of general importance.

Figure 2 illustrates the result of using the above adapter for the measurement of a compressor discharge plenum pressure. Of the two pressure signals shown in the photo, only the lower one was obtained with the adapter discussed above. A frequency estimation of the pressure oscillations in this signal indicates a good agreement with the first natural frequency (1582 Hz) of the adapter. Upon mounting the pressure transducer flush with the wall of the discharge plenum and repeating the measurement, the suspected resonance pressure oscillations disappeared as shown in Figure 3. Other adapters with resonant frequencies as high as 5 kHz were also tried; however, resonant conditions were again found to exist. Thus, for this application example it was considered essential to flush mount the pressure transducer. If this had not been possible, damping could have been introduced to eliminate the magnification of frequency components at or near the resonant frequency of the adapter.

CONCLUSIONS

1. The resonant frequency of pressure transducer adapter systems should be well above the highest frequency of interest in the measured pressure signal. Equation 11 gives an approximation for the adapter first natural frequency which is within 1.6% of the result predicted by an acoustic wave equation model.
2. As may be seen from Equation 11, the adapter resonant frequency may be increased by decreasing either the volume ratio, R, or the connecting tube length, L_2. Often the length, L_2, is fixed by geometrical constraints so that only R may be controlled.
3. The volume ratio, R, may be reduced by increasing the connecting tube area, S_2. However, since this also decreases the effective damping in the connecting passageway, an orifice such as that suggested in Reference 3 might be installed at the connecting tube end, $X_2 = L_2$, to provide both damping and good frequency response.
4. Pressure transducer adapter systems are likely to cause measurement errors in fixed geometry installations where numerous frequency components are possible due to the existence of standing waves.

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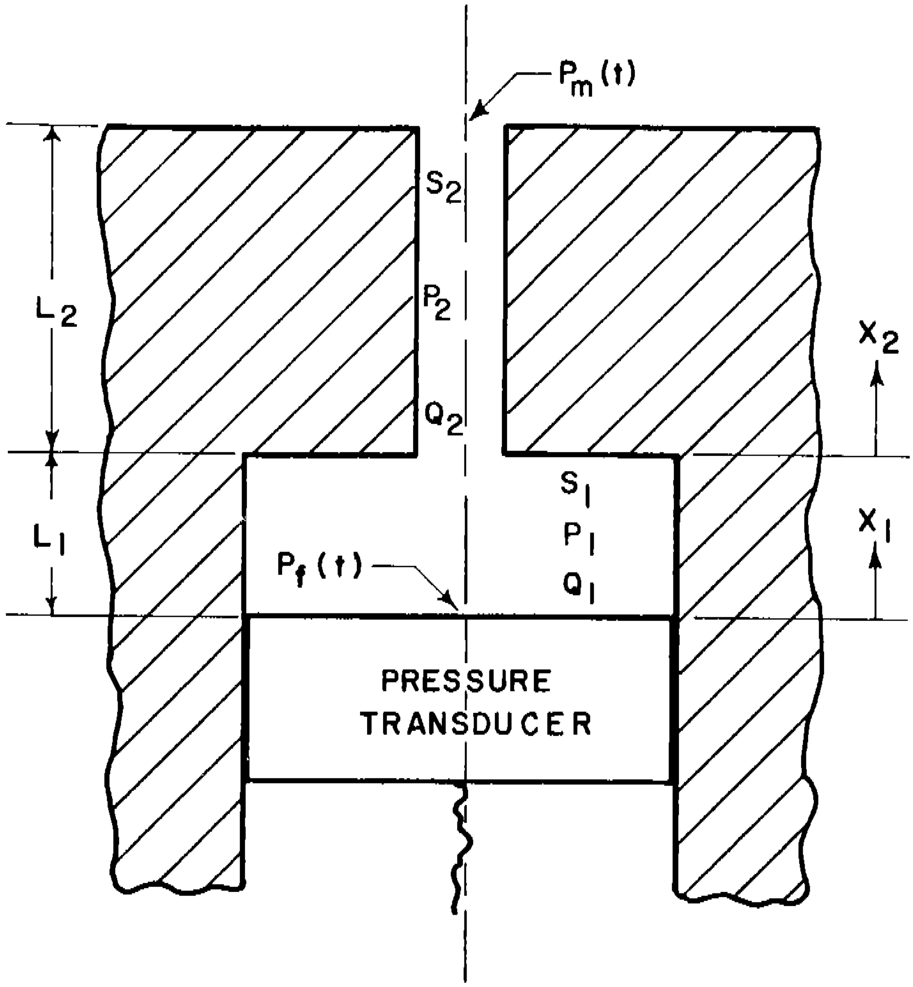

Figure 1. Basic Adapter System

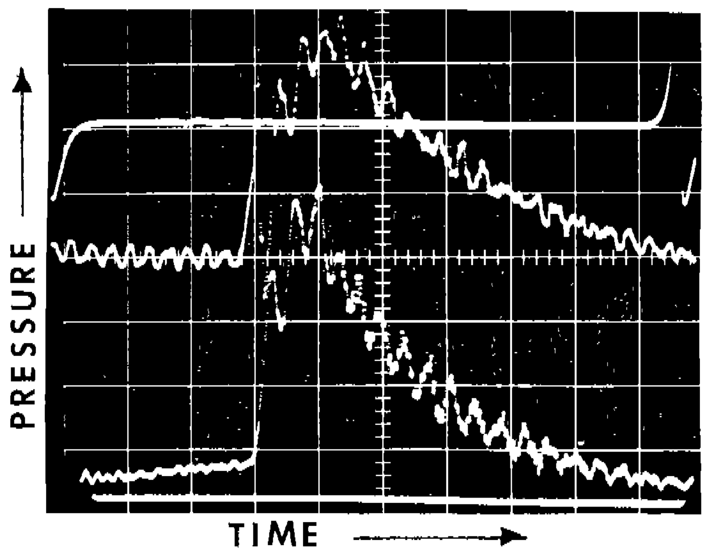

Figure 2. Compressor Plenum Pressure With Adapter

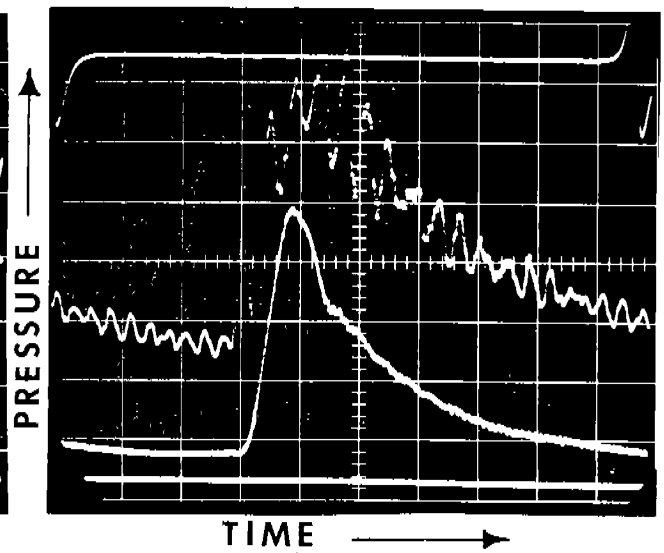

Figure 3. Compressor Plenum Pressure Without Adapter